

Tetrads in low-energy weak interactions

Alcides Garat¹

1. Instituto de Física, Facultad de Ciencias, Iguá 4225, esq. Mataojo, Montevideo, Uruguay.

(June 15th, 2006)

Tetrads are introduced in order to study the relationship between gravity and particle interactions, specially in weak processes at low energy. Through several examples like inverse Muon decay, elastic Neutrino-Electron scattering, it is explicitly shown how to assign to each vertex of the corresponding low-order Feynman diagram in a weak interaction, a particular set of tetrad vectors. The relationship between the tetrads associated to different vertices is exhibited explicitly to be generated by a $SU(2)$ local gauge transformation.

I. INTRODUCTION

We are trying to understand the underlying symmetries of different field architectures, by showing explicitly the local geometrical structure of different kinds of groups of transformations. In [1] we studied the local geometrical meaning of electromagnetic local gauge transformations. In [2] we studied the local geometrical meaning of $SU(2)$ local gauge transformations. Isomorphisms and homomorphisms were found that relate the standard groups of local gauge transformations with new groups of local geometrical transformations. These relationships can be explicitly displayed through the use of appropriately defined tetrads. It is the purpose of this work, to make use of already defined tetrads of different kinds [1] [2], in order to briefly show in an explicit way, the invariance of the metric tensor associated to a low-energy weak interaction, under different kinds of transformations. For instance, the invariance under electromagnetic local gauge transformations, the invariance under $SU(2)$ local gauge transformations, the invariance under local Lorentz transformations of the spinor fields [3] [4], etc. Since we are trying to “geometrize” the local gauge theories, it is interesting in its own right, to understand as well, the geometries that involve the standard fields associated with microparticle interactions. To that end, we introduce what we call “gravitational Feynman calculus”. We are able to explicitly show how to build a tetrad associated to a Feynman low-order diagram in low-energy weak interactions. In high energy interactions where virtual phenomena becomes relevant, a different approach is needed. We proceed to show how to assign a tetrad to each vertex, for instance in inverse Muon decay, and elastic Neutrino-Electron scattering. We strongly believe that the construction of tetrad fields, and metric tensors that explicitly display the local symmetries of microparticle interactions, are hinting us over a possible relationship or link, between General Relativity and Quantum Theories. We also demonstrate that it is possible to transform the tetrad associated to a vertex in a particular diagram to the tetrad assigned to another vertex in the same Feynman diagram through a local $SU(2)$ gauge transformation. Throughout the paper we use the conventions of [1] [2] [5]. In

particular we use a metric with sign conventions $-+++$.

II. GRAVITATIONAL FEYNMAN CALCULUS

It is of fundamental importance to understand the geometry of spacetime when particle interactions are taking place. Using the accumulated analysis for different kinds of gauge theories carried out in [1] [2], we are going to show explicitly how to assign to different Feynman diagrams in weakly interacting processes, different sets of tetrad vectors and therefore a metric tensor. The notation is a replica of the notation in [6], so we refer the reader to this reference. We also refer the reader to [6] [7] [8] for abundant literature citation, specially in the field of particle physics.

A. Weak interactions

The existence of mediators as it was shown in [1] [2] is irreplaceable as far as we are concerned with the construction of these kind of tetrads in weak interactions. In this case it is the existence of local $SU(2)$ “extremal” fields that allow us to build tetrads in weak processes. There are interactions involving the massive mediators where any virtual effect is negligible. For instance the W^- as the mediator in inverse Muon decay. The Z^0 mediator in elastic Neutrino-Electron scattering. This is important because the existence of virtual processes would require a different approach. We will analyze these processes through the use of appropriately defined tetrads.

1. Inverse Muon decay

Let us consider the process $e^-(1) + \nu_\mu(2) \rightarrow \nu_e(3) + \mu^-(4)$. There are two vertices. We invoke then the existence of the $SU(2)$ tetrads introduced in [2], specially the general tetrad structure presented in the section “Extremal field in $SU(2)$ geometrodynamics”. We called these general $SU(2)$ tetrad vectors $S_{(1)}^\mu \cdots S_{(4)}^\mu$. We briefly remind the reader about the structure of these latter tetrads,

$$S_{(1)}^\mu = \epsilon^{\mu\lambda} \epsilon_{\rho\lambda} X^\rho \quad (1)$$

$$S_{(2)}^\mu = \sqrt{-Q_{ym}/2} \epsilon^{\mu\lambda} X_\lambda \quad (2)$$

$$S_{(3)}^\mu = \sqrt{-Q_{ym}/2} * \epsilon^{\mu\lambda} Y_\lambda \quad (3)$$

$$S_{(4)}^\mu = * \epsilon^{\mu\lambda} * \epsilon_{\rho\lambda} Y^\rho, \quad (4)$$

where $\epsilon_{\mu\nu}$, is a local $SU(2)$ gauge invariant extremal tensor, and $Q_{ym} = \epsilon_{\mu\nu} \epsilon^{\mu\nu}$ [2]. There was a remaining freedom in the choice of two vector fields, X^ρ and Y^ρ . It is exactly through an appropriate choice for these two vector fields that we can identify a tetrad set for each vertex. In addition to the previously introduced notation and structures, let us call the non-null electromagnetic tetrads, following again the notation in [2], E_α^ρ . The indices α and β are reserved for locally inertial coordinate systems. Then, we can proceed to define for the first vertex the two vector fields,

$$X^\rho = Y^\rho = \bar{u}(3) \gamma^\alpha (\mathbf{1} - \gamma^5) u(1) E_\alpha^\rho . \quad (5)$$

We are basically associating to the first vertex a current $j_-^\alpha = \bar{u}(3) \gamma^\alpha (\mathbf{1} - \gamma^5) u(1)$, [6]. This current describes the process $e^- \rightarrow \nu_e + W^-$. For the second vertex we can choose for instance,

$$X^\rho = Y^\rho = \bar{u}(4) \gamma^\alpha (\mathbf{1} - \gamma^5) u(2) E_\alpha^\rho . \quad (6)$$

Again, we are assigning to the second vertex a current $j_-^\alpha = \bar{u}(4) \gamma^\alpha (\mathbf{1} - \gamma^5) u(2)$, [6] describing the process $\nu_\mu + W^- \rightarrow \mu^-$. It is evident from all the analysis in [2] that the geometrical transition from vertex one to vertex two and vice-versa, is an $SU(2)$ generated local gauge transformation. That is only allowed through the existence of massive mediators. Following the ideas in [2] we can start by choosing for instance,

$$X_\rho = Tr[\Sigma^{\alpha\beta} E_\alpha^\sigma E_\beta^\lambda * \xi_{\sigma\rho} * \xi_{\lambda\tau} A^\tau] \quad (7)$$

$$Y_\rho = Tr[\Sigma^{\alpha\beta} E_\alpha^\sigma E_\beta^\lambda * \xi_{\sigma\rho} * \xi_{\lambda\tau} * A^\tau] . \quad (8)$$

The $\Sigma^{\alpha\beta}$ objects are analyzed in appendix II in reference [2], $\xi_{\sigma\rho}$ are the electromagnetic “extremal” fields introduced in [1], etc. Through a local $SU(2)$ gauge transformation on blade one, we can “rotate” the tetrad vectors on blade one, until X^ρ in (7) becomes X^ρ in (5). We can also “rotate” the tetrad vectors on blade two, until Y^ρ in (8) becomes Y^ρ in (5). Again we can start with (7) and appropriately “rotate” the tetrad vectors on blade one, until they become the ones corresponding to X^ρ given in (6). Similar for Y^ρ in this second case. It is evident then that (5) and (6) are connected through local $SU(2)$ gauge transformations on blades one and two, that in turn, leave invariant the metric tensor.

We can also notice that the vector fields (5-6) are not strictly vectors but pseudovectors under local parity transformations, see [6]. But the metric tensor remains unaltered under these local parity transformations. It is as if the geometry associated to the $e^-(1)$ and $\nu_e(3)$ can be transformed through the existence of a massive mediator into the geometry associated to the $\nu_\mu(2)$ and $\mu^-(4)$.

2. Elastic Neutrino-Electron scattering

Now, we are considering neutral currents. In particular the interaction process $\nu_\mu(1) + e^-(2) \rightarrow \nu_\mu(3) + e^-(4)$. As before we can assign to the first vertex the choice,

$$X^\rho = Y^\rho = \bar{u}(3) \gamma^\alpha (\mathbf{1} - \gamma^5) u(1) Z_\alpha^\rho . \quad (9)$$

The current $j_-^\alpha = \bar{u}(3) \gamma^\alpha (\mathbf{1} - \gamma^5) u(1)$, represents the process $\nu_\mu(1) \rightarrow \nu_\mu(3) + Z^0$. The tetrad Z_α^ρ is built as follows. Following again the notation in [6] we know we have available a local vector field Z_μ that results from the Weinberg rotation through the angle θ_w , in addition to the standard electromagnetic vector field A_μ . The rotation can be written,

$$A_\mu = B_\mu \cos \theta_w + W_\mu^3 \sin \theta_w \quad (10)$$

$$Z_\mu = -B_\mu \sin \theta_w + W_\mu^3 \cos \theta_w . \quad (11)$$

The electro-weak mixing involves a weak isotriplet of intermediate vector bosons \mathbf{W} coupled to three weak isospin currents, and an isosinglet intermediate vector boson B_μ coupled to the weak hypercharge current. If we follow all the steps in [1], we can build out of the curl $Z_{\mu;\nu} - Z_{\nu;\mu}$, a new tetrad. The symbol “;” stands for the usual covariant derivative associated with the metric tensor $g_{\mu\nu}$. This tetrad would once more involve the choice of two vector fields, see [1]. We can choose for instance Z_μ and B_μ as these two vector fields. Then, the tetrad that couples to the neutrino current is associated to the massive Z^0 .

The second vertex could be assigned a choice,

$$X^\rho = Y^\rho = \bar{u}(4) \gamma^\alpha (\mathbf{c}_V - \mathbf{c}_A \gamma^5) u(2) E_\alpha^\rho , \quad (12)$$

representing $e^-(2) + Z^0 \rightarrow e^-(4)$. For this particular interaction $c_V = -\frac{1}{2} + 2 \sin \theta_w$, and $c_A = -\frac{1}{2}$, where θ_w is again the Weinberg angle, [6]. The massive mediator allows again for a $SU(2)$ local gauge transformation between the tetrad vectors chosen for vertex one and the ones chosen for vertex two. The neutral current works as a geometry mediator between the scattered particles.

III. CONCLUSIONS

We have explored the possibility of assigning tetrads to Feynman diagrams. Having done this explicitly, a number of questions naturally arise. We want these concluding remarks to be a summary of these open questions.

- The order of the formulations. We have worked out the low-order diagrams. Then, what happens with higher order diagrams ?. The tetrads admit the choice of two vector fields, and the higher order are additive exactly as in the quantum theories, in these vector fields available as a choice. But there is more to understand. Do the higher order diagrams represent contributions coming from higher order perturbative theories of a full relativistic formulation of these interactions, for instance ?.
- The issue of “gauge gravity”. Since in [1] [2] it was explicitly proved that the Abelian and non-Abelian gauge theories represent special symmetries of the gravitational field, we can ask about the meaning of “gauge gravity”. The electromagnetic field is associated to the $LB1$ and $LB2$ symmetries of the gravitational field. The $SU(2)$ group of local gauge transformations is associated to the symmetries of the tensor product of three $LB1$ or three $LB2$ groups of transformations. Analogous for $SU(3)$. Then, it is not obvious to understand what is the meaning of a statement like, “casting the theory of gravity into a Yang-Mills formulation”.
- The issue of quantum gravity. It has been proved explicitly that metric tensors can be associated with microparticle interactions. These constructions are possible by means of non-null Abelian fields, and by means of $SU(2)$

local non-Abelian fields. The quantum is connected through the existence of these tetrad fields to gravity. The tetrads of different nature that we were able to build in [1] [2] and the present work, establish a link between the standard locally inertial flat field environment of the traditional standard quantum theories in weak interactions on one hand, and the curved spacetime of gravity on the other hand. The point is the following, why are we using in quantum gravity similar conceptual foundations to theories that are not formulated in curved spacetimes ?.

- The issue of the Higgs mechanism. It is a device conceived in its relationship with the nature of mass, for instance of the mass mediators. In the present tetrad and gravitational environment we can ask if it is necessary, or the mass comes into existence due to the presence of gravity ?.
- The issue of symmetry-breaking. It was proved in manuscripts [1] [2] that the tetrads built along the lines of expressions (1-4) are invariant under local electromagnetic gauge transformations, and local $SU(2)$ gauge transformations. This result was valid for three independent sets of tetrads [2]. But when assigning a tetrad set to a vertex in a low-energy weak process diagram, we are making a particular choice for the two vectors X^ρ and Y^ρ . For instance, through associated currents we are choosing a particular gauge, and a different one for each vertex, like in inverse Muon decay or elastic Neutrino-Electron scattering. Then, we wonder if this gauge fixing procedure could be the geometrical form of the standard symmetry-breaking process. Hereby, we can see that it is gravity the field that bridges the two gauges associated to the two vertices, through a local $SU(2)$ gauge transformation, that in turn, leaves invariant the metric tensor.

ACKNOWLEDGMENTS

I am grateful to R. Gambini and J. Pullin for reading this manuscript and for many fruitful discussions.

-
- [1] A. Garat, J. Math. Phys. **46**, 102502 (2005).
 - [2] A. Garat, gr-qc/0602049.
 - [3] M. Kaku, *Quantum Field Theory: A Modern Introduction* (Oxford University Press, 1993).
 - [4] L. Álvarez-Gaumé and M. A. Vázquez-Mozo, *Introductory Lectures on Quantum Field Theory* (arXiv:hep-th/0510040).
 - [5] C. Misner and J. A. Wheeler, Annals of Physics **2**, 525 (1957).
 - [6] D. Griffiths , *Introduction to elementary particles* (John Wiley & Sons, Inc. , 1987).
 - [7] T. P. Cheng and L. F. Li , *Gauge Theory of Elementary Particle Physics* (Oxford University Press, 1989).

[8] W. Greiner and B. Mueller, *Gauge Theory of Weak Interactions* (Springer Verlag Gmbh, 1996).